UNDERVISNINGS
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Mathematical Formula Booklet

stx
B-level

May 2020

Denne udgave af Matematisk formelsamling
stx B-niveau er udgivet af Undervisningsministeriet
og gjort tilgængelig på uvm.dk.

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Version May 2020

Preface:

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"Mathematical formula collection stx B" has been prepared for use by the students at the written exam and in the teaching of mathematics at the stx B level.

The formula collection contains the topics that appear in the mathematics curriculum at stx B-level within both core content and supplementary content.

For the purpose of overview, formulas for area and volume of elementary geometric figures are included.

Furthermore, the formula collection contains a list of standard mathematical symbols.

The purpose of this is to give students a quick overview and to contribute to guide teachers and authors of educational material to use uniform notation, symbol language and terminology. The list of standard mathematical symbols therefore goes beyond the core content but remains within the mathematical universe at the upper secondary level (stx and hf).

A few of the formulas in the formula collection are only applicable under some conditions (e.g., that the denominator of a fraction is different from 0). For reasons of clarity such assumptions are not explicitly mentioned.

The figures are included as an illustration of the formulas. The figure often shows only one case among several possible cases.

The meaning of the quantities appearing in the formulas is not always explained. However, in cases where the meaning does not follow immediately by customary usage an explanation will be given.

Birte Iversen
Ministry of Education,
The Board of Education and Quality,
Office of Examinations, Exams and Tests
May 2020

Contents

Percentage and interest

Present value B
Future value S

(1) $S = B \cdot (1 + r)$

Relative change,
growth rate r

(2) $r = \frac{S}{B} - 1$

Percentage change p

(3) $p\,\% = r \cdot 100\,\%$

Compound interest formula
Principal K_0
Interest rate p % per period
Future value K after n periods

(4) $K = K_0 \cdot (1 + r)^n$, where $r = \frac{p}{100}$

Annuity savings
Deposit every period b
Interest rate r
Number of deposits n
Future value A after last deposit

(5) $A = b \cdot \frac{(1 + r)^n - 1}{r}$

Annuity loan
Principal G
Interest rate r
Number of payments n
Periodic payment y

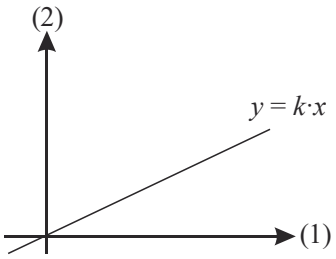
(6) $y = G \cdot \frac{r}{1 - (1 + r)^{-n}}$

Index values

Value	B	S
Index value	I_B	I_S

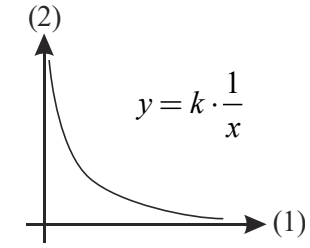
(7) $I_S = \frac{S}{B} \cdot I_B \qquad S = \frac{I_S}{I_B} \cdot B$

Proportionality



x and y are proportional
Proportionality constant k

(8) $y = k \cdot x$ $\qquad \frac{y}{x} = k$



x and y are inversely proportional

(9) $y = k \cdot \frac{1}{x}$ $\qquad x \cdot y = k$

Fraction rules

(10) $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

(11) $\frac{a}{\frac{b}{c}} = \frac{a \cdot c}{b}$

(12) $\frac{\frac{a}{b}}{c} = \frac{a}{b \cdot c}$

(13) $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$

(14) $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Square expansions

(15) $(a+b)^2 = a^2 + b^2 + 2a \cdot b$

(16) $(a-b)^2 = a^2 + b^2 - 2a \cdot b$

(17) $(a+b)(a-b) = a^2 - b^2$

Powers and roots

(18) $a^r \cdot a^s = a^{r+s}$

(19) $\frac{a^r}{a^s} = a^{r-s}$

(20) $(a^r)^s = a^{r \cdot s}$

(21) $(a \cdot b)^r = a^r \cdot b^r$

(22) $\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$

(23) $a^0 = 1$

(24) $a^{-r} = \frac{1}{a^r}$

(25) $a^{-1} = \frac{1}{a}$

(26) $\sqrt[r]{a} = a^{\frac{1}{r}}$

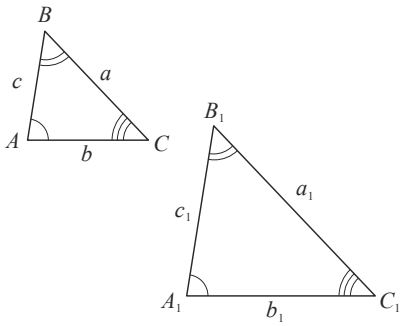
(27) $\sqrt[s]{a^r} = a^{\frac{r}{s}}$

(28) $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

(29) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(30) $\sqrt{a} = a^{\frac{1}{2}}$

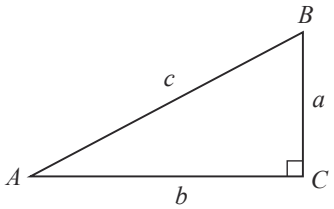
Similar triangles



(31) $$\frac{a_1}{a} = \frac{b_1}{b} = \frac{c_1}{c} = k$$

(32) $$a_1 = k \cdot a$$
$$b_1 = k \cdot b$$
$$c_1 = k \cdot c$$

Right-angled triangle



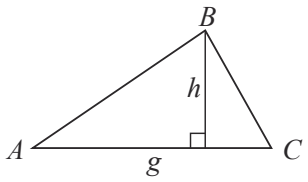
The Pythagorean theorem (33) $$c^2 = a^2 + b^2$$

cosine (34) $$\cos(A) = \frac{b}{c}$$

sine (35) $$\sin(A) = \frac{a}{c}$$

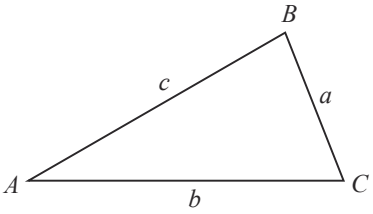
tangent (36) $$\tan(A) = \frac{a}{b}$$

Triangles



Sum of angles of a triangle (37) $A+B+C=180°$

Area T of a triangle (38) $T=\frac{1}{2}h\cdot g$

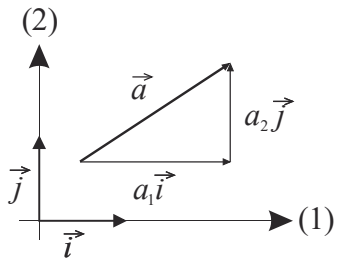


Law of cosines (39) $c^2=a^2+b^2-2a\cdot b\cdot\cos(C)$

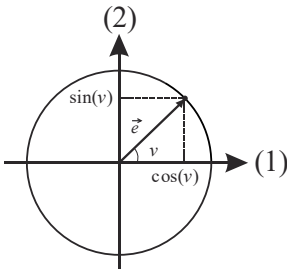
Law of sines (40) $\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$

Area T of a triangle (41) $T=\frac{1}{2}a\cdot b\cdot\sin(C)$

Vectors in 2D

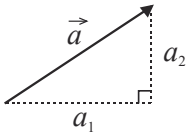


Coordinates of vector $\vec{a}$, where $|\vec{i}|=|\vec{j}|=1$ (42) $\vec{a}=a_1\cdot\vec{i}+a_2\cdot\vec{j}=\begin{pmatrix}a_1\\a_2\end{pmatrix}$

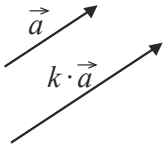


Unit vector (43) $\vec{e}=\begin{pmatrix}\cos(v)\\\sin(v)\end{pmatrix}$

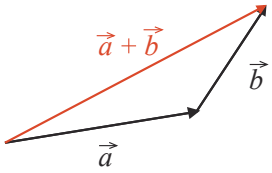
Unit vector $\vec{e}$ in the same direction as $\vec{a}$ (44) $\vec{e}=\frac{\vec{a}}{|\vec{a}|}$



Length (norm) of vector $\vec{a}$ (45) $|\vec{a}|=\left|\begin{pmatrix}a_1\\a_2\end{pmatrix}\right|=\sqrt{a_1{}^2+a_2{}^2}$

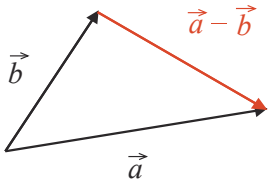


Vector $\vec{a}$ multiplied by a scalar k (46) $k\cdot\vec{a}=k\cdot\begin{pmatrix}a_1\\a_2\end{pmatrix}=\begin{pmatrix}k\cdot a_1\\k\cdot a_2\end{pmatrix}$



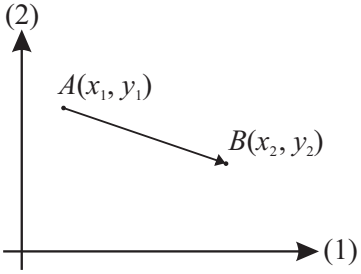
Sum of two vectors

(47) $$\vec{a}+\vec{b}=\begin{pmatrix}a_1\\a_2\end{pmatrix}+\begin{pmatrix}b_1\\b_2\end{pmatrix}=\begin{pmatrix}a_1+b_1\\a_2+b_2\end{pmatrix}$$



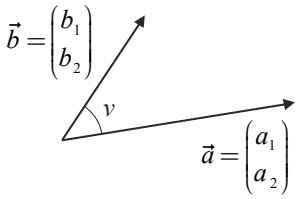
Difference between two vectors

(48) $$\vec{a}-\vec{b}=\begin{pmatrix}a_1\\a_2\end{pmatrix}-\begin{pmatrix}b_1\\b_2\end{pmatrix}=\begin{pmatrix}a_1-b_1\\a_2-b_2\end{pmatrix}$$



Coordinates of vector $\overrightarrow{AB}$

(49) $$\overrightarrow{AB}=\begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix}$$



Scalar product (dot product) of $\vec{a}$ and $\vec{b}$

(50) $$\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2$$

(51) $$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos(v)$$

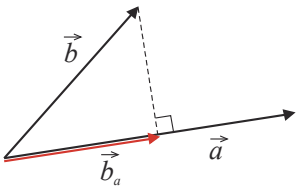
(52) $$\cos(v)=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

Perpendicular vectors

(53) $$\vec{a}\cdot\vec{b}=0 \Leftrightarrow \vec{a}\perp\vec{b}$$

Squaring a vector

(54) $$\vec{a}\cdot\vec{a}=\vec{a}^2=|\vec{a}|^2$$

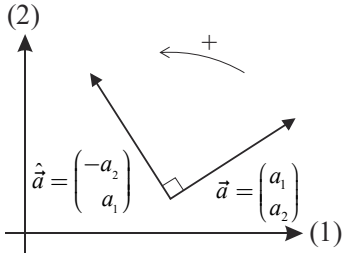


Projection of $\vec{b}$ onto $\vec{a}$

(55) $\vec{b}_a = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\cdot\vec{a}$

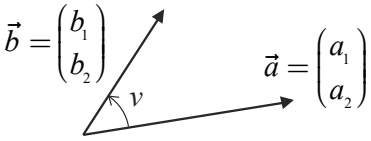
The length of the projection vector

(56) $|\vec{b}_a| = \frac{|\vec{a}\cdot\vec{b}|}{|\vec{a}|}$



Perpendicular vector to $\vec{a}$

(57) $\hat{\vec{a}} = \begin{pmatrix} \widehat{a_1} \\ a_2 \end{pmatrix} = \begin{pmatrix} -a_2 \\ a_1 \end{pmatrix}$



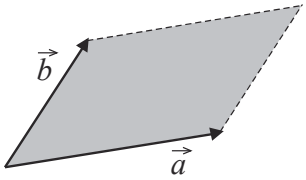
The determinant of a pair of vectors $(\vec{a},\vec{b})$

(58) $\det(\vec{a},\vec{b}) = \hat{\vec{a}}\cdot\vec{b} = a_1b_2 - a_2b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

(59) $\det(\vec{a},\vec{b}) = |\vec{a}|\cdot|\vec{b}|\cdot\sin(v)$

Parallel vectors

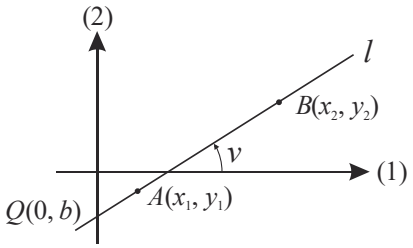
(60) $\det(\vec{a},\vec{b}) = 0 \quad \Leftrightarrow \quad \vec{a}\,||\,\vec{b}$



Area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$

(61) $A = |\det(\vec{a},\vec{b})|$

Lines, circles and parabolas



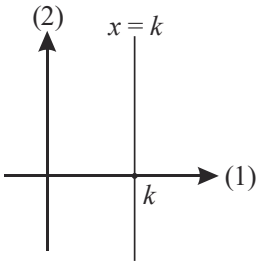
Equation of the line l through $Q(0,b)$ with slope a (62) $y = a \cdot x + b$

Slope (gradient) a of the line l passing through $A(x_1, y_1)$ and $B(x_2, y_2)$ (63) $a = \frac{y_2 - y_1}{x_2 - x_1}$

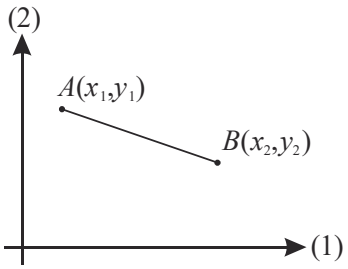
y-intercept (64) $b = y_1 - a \cdot x_1$

Equation of the line l through $A(x_1, y_1)$ of slope a (65) $y = a \cdot (x - x_1) + y_1$

Angle v of inclination is the angle from the x-axis to l (positive or negative) (66) $a = \tan(v)$

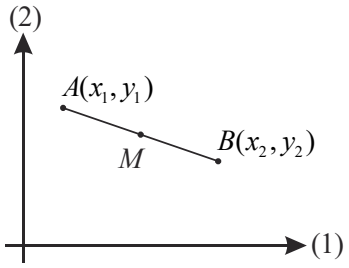


Equation of a vertical line (67) $x = k$



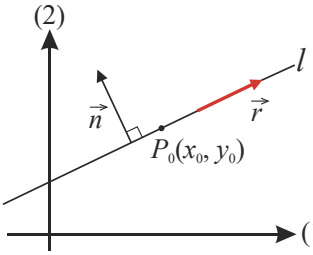
Distance $|AB|$ between two points $A(x_1, y_1)$ and $B(x_2, y_2)$

(69) $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Midpoint M of line segment AB

(70) $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

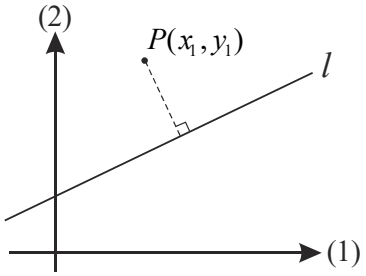


Equation of the line l passing through P_0 having normal vector $\vec{n} = \begin{pmatrix} a \\ b \end{pmatrix}$

(71) $a \cdot (x - x_0) + b \cdot (y - y_0) = 0$

Parametric equations of the line l passing through P_0 having direction vector $\vec{r} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$

(72) $$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + t \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$$

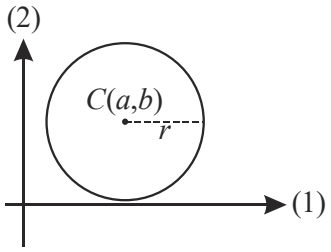


Distance dist(P,l) of the point $P(x_1, y_1)$ to the line l with equation $y = a \cdot x + b$

(73) $$\text{dist}(P,l) = \frac{|a \cdot x_1 + b - y_1|}{\sqrt{a^2 + 1}}$$

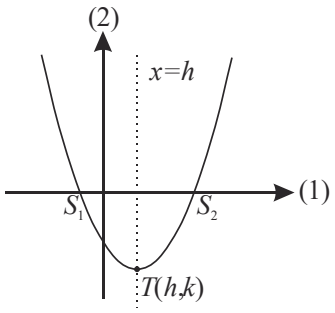
Distance dist(P,l) of the point $P(x_1, y_1)$ to the line l with equation $a \cdot x + b \cdot y + c = 0$

(74) $$\text{dist}(P,l) = \frac{|a \cdot x_1 + b \cdot y_1 + c|}{\sqrt{a^2 + b^2}}$$



Equation of circle with center $C(a,b)$ and radius r

(75) $$(x-a)^2 + (y-b)^2 = r^2$$



Equation of parabola with symmetry axis parallel to the *y*-axis

(76) $y = a \cdot x^2 + b \cdot x + c = a \cdot (x - h)^2 + k$

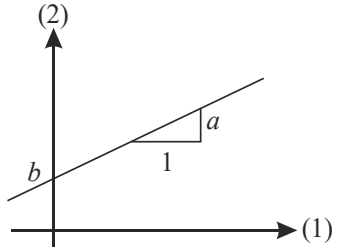
Vertex T

(77) $T(h,k) = T\left(\frac{-b}{2a}, \frac{-d}{4a}\right), \quad d = b^2 - 4ac$

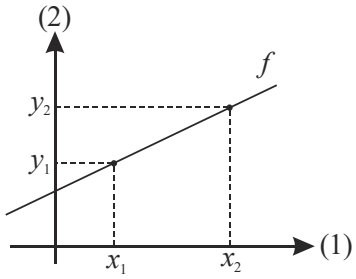
Points of intersection S_1 og S_2 with the *x*-axis

(78) $S_1\left(\frac{-b - \sqrt{d}}{2a}, 0\right), \; S_2\left(\frac{-b + \sqrt{d}}{2a}, 0\right)$

Linear functions



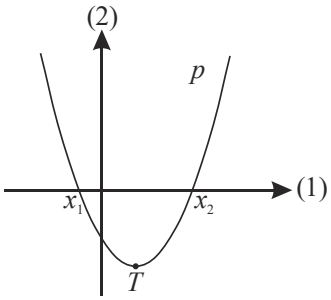
First degree polynomial, linear function f (79) $f(x) = a \cdot x + b$



Slope (gradient) a from two points on the line between (x_1, y_1) and (x_2, y_2) (80) $a = \dfrac{y_2 - y_1}{x_2 - x_1}$

y-intercept (81) $b = y_1 - a \cdot x_1$

Quadratic functions

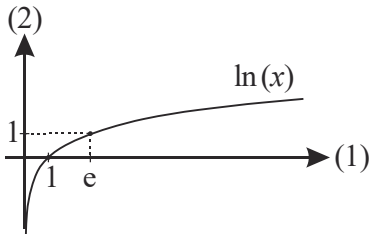


Quadratic function (second degree polynomial) p with zeros (roots) x_1 and x_2 (82) $p(x) = a \cdot x^2 + b \cdot x + c$
$= a \cdot (x - x_1) \cdot (x - x_2)$

Zeros (roots) of p (83) $x_1 = \dfrac{-b - \sqrt{d}}{2a}$, $x_2 = \dfrac{-b + \sqrt{d}}{2a}$,
where $d = b^2 - 4ac$

Vertex T (84) $T\left(\dfrac{-b}{2a}, \dfrac{-d}{4a}\right)$

Logarithms



The graph of the natural logarithm

(85) $\ln(x) \to -\infty \quad \text{as} \quad x \to 0$

(86) $\ln(x) \to \infty \quad \text{as} \quad x \to \infty$

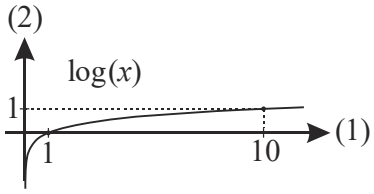
(87) $y = \ln(x) \quad \Leftrightarrow \quad x = \mathrm{e}^{y}$

(88) $\ln(\mathrm{e}) = 1$

(89) $\ln(a \cdot b) = \ln(a) + \ln(b)$

(90) $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

(91) $\ln(a^{r}) = r \cdot \ln(a)$



The graph of the logarithmic with base 10

(92) $\log(x) \to -\infty \quad \text{as} \quad x \to 0$

(93) $\log(x) \to \infty \quad \text{as} \quad x \to \infty$

(94) $y = \log(x) \quad \Leftrightarrow \quad x = 10^{y}$

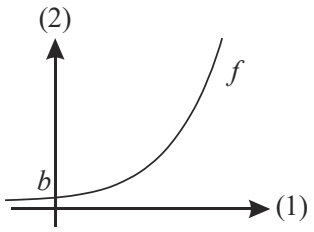
(95) $\log(10) = 1$

(96) $\log(a \cdot b) = \log(a) + \log(b)$

(97) $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

(98) $\log(a^{r}) = r \cdot \log(a)$

Exponentially increasing functions



The graph of an exponentially increasing function f
$a > 1$
growth rate $r > 0$
$k > 0$

(99) $$f(x) = b \cdot a^x = b \cdot (1+r)^x = b \cdot \mathrm{e}^{k \cdot x}, \text{ where } k = \ln(a)$$

(100) $$f(x) \to \infty \quad \text{as} \quad x \to \infty$$

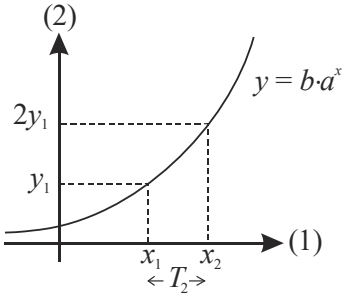
(101) $$f(x) \to 0 \quad \text{as} \quad x \to -\infty$$

Multiplication factor a
given two points on the graph
(x_1, y_1) and (x_2, y_2)

(102) $$a = \sqrt[x_2 - x_1]{\frac{y_2}{y_1}} = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

y-intercept

(103) $$b = \frac{y_1}{a^{x_1}}$$

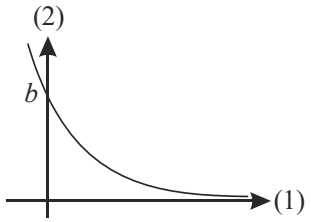


Doubling constant T_2

(104) $$T_2 = x_2 - x_1$$

(105) $$T_2 = \frac{\log(2)}{\log(a)} = \frac{\ln(2)}{\ln(a)} = \frac{\ln(2)}{k}$$

Exponentially decreasing functions



The graph of an exponentially decreasing function f
$0 < a < 1$
growth rate $r < 0$
$k < 0$

(106) $$f(x) = b \cdot a^x = b \cdot (1+r)^x = b \cdot \mathrm{e}^{k \cdot x}, \quad \text{where } k = \ln(a)$$

(107) $$f(x) \to 0 \quad \text{as} \quad x \to \infty$$

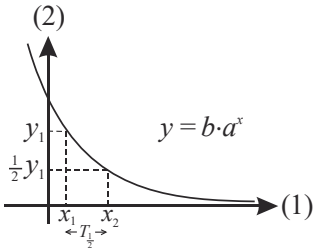
(108) $$f(x) \to \infty \quad \text{as} \quad x \to -\infty$$

Multiplication factor a using two points on the graph (x_1, y_1) and (x_2, y_2)

(109) $$a = \sqrt[x_2 - x_1]{\frac{y_2}{y_1}} = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

y-intercept

(110) $$b = \frac{y_1}{a^{x_1}}$$



Halving constant (half-life) $T_{\frac{1}{2}}$

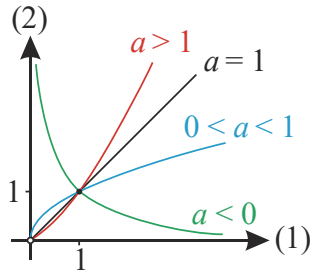
(111) $$T_{\frac{1}{2}} = x_2 - x_1$$

(112) $$T_{\frac{1}{2}} = \frac{\log\left(\frac{1}{2}\right)}{\log(a)} = \frac{\ln\left(\frac{1}{2}\right)}{\ln(a)} = \frac{\ln\left(\frac{1}{2}\right)}{k}$$

Power functions

Power function

(113) $f(x) = b \cdot x^a$



Graphs of $f(x) = x^a$

Determining the number a given two points on the graph (x_1, y_1) and (x_2, y_2)

(114) $a = \dfrac{\log(y_2) - \log(y_1)}{\log(x_2) - \log(x_1)} = \dfrac{\ln(y_2) - \ln(y_1)}{\ln(x_2) - \ln(x_1)}$

(115) $b = \dfrac{y_1}{x_1^a}$

Multiplying x by a factor $1 + r_x$ will result in a multiplication of $f(x)$ by a factor $1 + r_y$

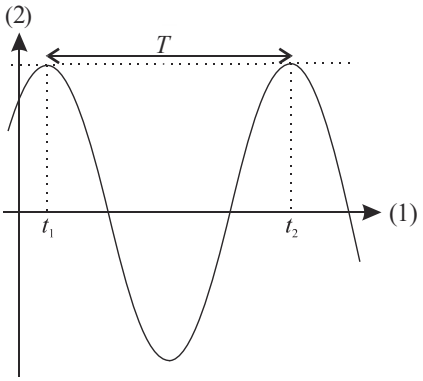
(116) $1 + r_y = (1 + r_x)^a$

Multiplying x by a factor k will result in a multiplication of $f(x)$ by a factor k^a

(117) $f(k \cdot x) = k^a \cdot f(x)$

Trigonometric functions

Sinusoidal function f (118) $f(t) = A \cdot \sin(\omega \cdot t + \varphi)$



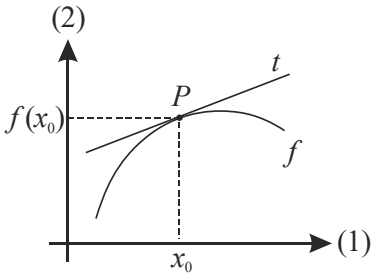
Graph of a sinusoidal function f with amplitude A and period T

(119) $T = t_2 - t_1 = \frac{2\pi}{\omega}$

Differential Calculus

Derivative (differential quotient) $f'(x_0)$ of the function f at x_0

(120) $$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$



Equation of the tangent line t to the graph of f at $P(x_0, f(x_0))$

(121) $$y = f'(x_0) \cdot (x - x_0) + f(x_0)$$
or
$y = a \cdot x + b$, where
$a = f'(x_0)$ and $b = y_0 - a \cdot x_0$

Rules of differentiation

(122) $$(k \cdot f(x))' = k \cdot f'(x)$$

(123) $$(f(x) + g(x))' = f'(x) + g'(x)$$

(124) $$(f(x) - g(x))' = f'(x) - g'(x)$$

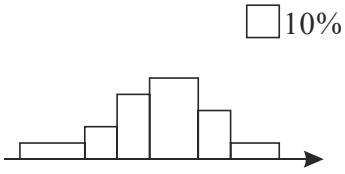
(125) $$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

(126) $$(f(a \cdot x + b))' = a \cdot f'(a \cdot x + b)$$

Derivatives

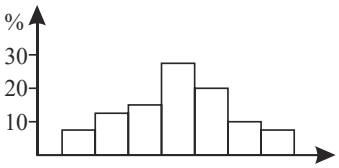
		Function $y = f(x)$	Derivative $y' = f'(x) = \frac{dy}{dx}$
Linear function	(127)	$a \cdot x + b$	a
	(128)	k	0
The natural logarithm	(129)	$\ln(x)$	$\frac{1}{x} = x^{-1}$
Exponential functions	(130)	e^{x}	e^{x}
	(131)	$e^{k\,x}$	$k \cdot e^{k\,x}$
	(132)	a^{x}	$a^{x} \cdot \ln(a)$
Power functions	(133)	x^{a}	$a \cdot x^{a-1}$
	(134)	$\frac{1}{x} = x^{-1}$	$-\frac{1}{x^{2}} = -x^{-2}$
	(135)	$\sqrt{x} = x^{\frac{1}{2}}$	$\frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$
Trigonometric functions	(136)	$\cos(x)$	$-\sin(x)$
	(137)	$\sin(x)$	$\cos(x)$

Grouped observations



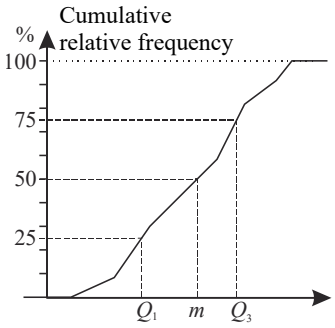
Histogram

(138) Area of a rectangle corresponds to the frequency for the interval



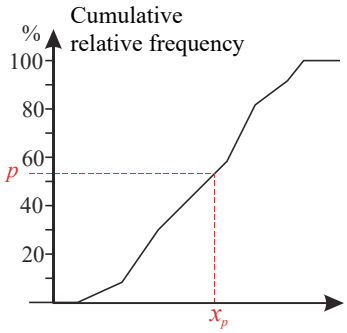
Histogram
with *equal* intervals

(139) Height of a rectangle corresponds to the frequency for the interval



Cumulative curve

(140) Q_1 : lower quartile, 25% -fractile
m : median, 50% -fractile
Q_3 : upper quartile, 75% -fractile
x_p : p% -fractile



Ungrouped observations

Dot plot

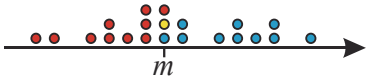
(141) Observations on a number line

(142) *min*: minimum

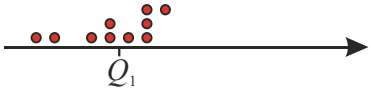
(143) *max*: maximum

Range

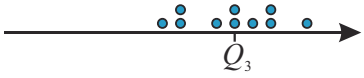
(144) $max - min$



(145) *m*: median
(middle observation, when the number of observations is odd, otherwise the average of the two middle observations)



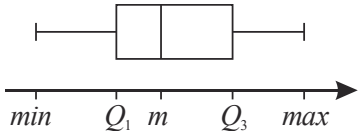
(146) Q_1: lower quartile
(median of the lower half of the observations)



(147) Q_3: upper quartile
(median of the upper half of the observations)

Interquartile range

(148) $Q_3 - Q_1$



(149) Boxplot, box-and-whiskers plot
(box height is irrelevant)

Quartile set

(150) (Q_1, m, Q_3)

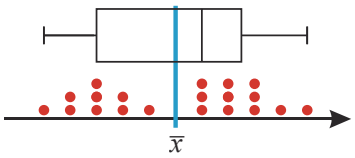
Five-number summary

(151) (min, Q_1, m, Q_3, max)

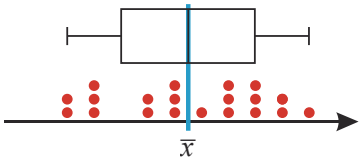
Outlier (152) An observation more than one and a half times the interquartile range below the lower quartile or more than one and a half times the interquartile range above the upper quartile

Mean $\bar{x}$ of the observations $x_1, x_2, \ldots, x_n$ (153) $$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

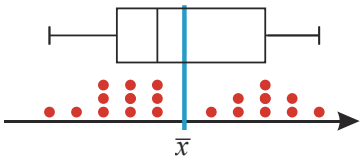
Standard deviation of observations $x_1, x_2, \ldots, x_n$ (154) $$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}} = \sqrt{\frac{(x_1 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$



Left-skew distribution (155) Mean is less than median $\bar{x} < m$



Non-skew distribution (156) Mean equals median $\bar{x} = m$



Right-skew distribution (157) Mean is greater than median $\bar{x} > m$

Estimating the population mean and standard deviation using a random sample $x_1, x_2, \ldots, x_n$

Estimate $\bar{x}$ of the mean (153a) $$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

Estimate s of the standard deviation (154a) $$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(x_1 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n-1}}$$

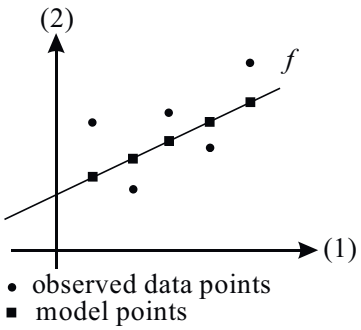
Linear regression

Table with bivariate observed data (158)

x	x_1	x_2	x_3	…	x_n
y	y_1	y_2	y_3	…	y_n

Regression line (159) Line of best fit, graph of $f(x) = a \cdot x + b$

Scatter plot and line of best fit (160)

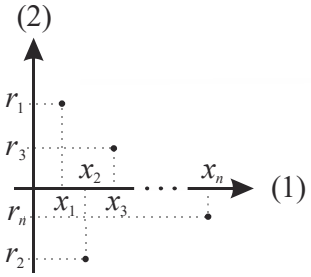


Residual (161) Difference between observed y-value and the corresponding model y-value

Table of residuals (162)

x	x_1	x_2	…	x_n
Residual	$r_1 = y_1 - f(x_1)$	$r_2 = y_2 - f(x_2)$	…	$r_n = y_n - f(x_n)$

Residual plot (163)



Residual standard deviation (164) $s = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n-2}}$

Combinatorics

Multiplication rule
Number of ways to choose *both* one element of N *and* one element of M, where N has n elements and M has m elements

(165) $n \cdot m$

Addition rule
Number of ways to choose *either* one element of N *or* one element of M, where N has n elements and M has m elements

(166) $n+m$

Factorial

(167) $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$

Permutations
Number of ways to select r elements among n elements, when order matters

(168) $P(n,r) = \frac{n!}{(n-r)!}$

Combinations
Number of ways to select r elements among n elements, when order does not matter

(169) $K(n,r) = \frac{n!}{r!(n-r)!}$

Probability

Probability space with sample space U and probabilities p (170) (U, p)

Sample space U with n outcomes (171) Set of all outcomes $\{u_1, u_2, \cdots, u_n\}$

Sum of all probabilities (172) $p_1 + p_2 + p_3 + ... + p_n = 1$

Table of probabilities (173)

Outcome	u_1	u_2	u_3	…	u_n
Probability	p_1	p_2	p_3	…	p_n

Event A with k outcomes from U (174) Set of k outcomes from U

Probability of event A (175) Sum of the probabilities of the k outcomes

Uniform probability space

All outcomes have the same probability (176) $p_1 = p_2 = p_3 = ... = p_n = \frac{1}{n}$

Probability of selecting an element from A (177) $P(A) = \frac{k}{n}$

$$= \frac{number\ of\ favourable\ outcomes\ to\ A}{number\ of\ possible\ outcomes}$$

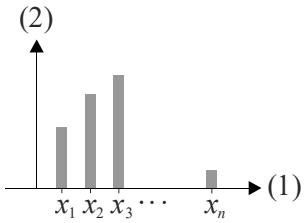
Probability of *both A and B*, when A and B are independent events (178) $P(\text{both } A \text{ and } B) = P(A) \cdot P(B)$

Probability of *A or B*, when A and B are mutually exclusive events (179) $P(A \text{ or } B) = P(A) + P(B)$

Probability distribution table for a random variable X (180)

x_i	x_1	x_2	x_3	…	x_n
$P(X = x_i)$	p_1	p_2	p_3	…	p_n

Bar chart. Height of a bar corresponds to the probability of the outcome (181)



Mean (mean value) of a random variable X (182)

$$\mu = E(X) = \sum_{i=1}^{n} x_i \cdot P(X = x_i)$$
$$= x_1 \cdot p_1 + x_2 \cdot p_2 + x_3 \cdot p_3 + \cdots + x_n \cdot p_n$$

Variance of a random variable X (183)

$$\mathrm{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot P(X = x_i)$$
$$= (x_1 - \mu)^2 \cdot p_1 + \cdots + (x_n - \mu)^2 \cdot p_n$$

Standard deviation of random variable X (184)

$$\sigma = \sigma(X) = \sqrt{\mathrm{Var}(X)}$$

Binomial distribution

Binomially distributed random variable X with parameters n (number of trials) and p (probability of "success") (185)

$$X \sim b(n, p)$$

Binomial coefficient $K(n,r)$ (186)

$$K(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(187)

$$K(n,r) = K(n,n-r)$$

Probability distribution for a binomially distributed random variable X (188)

$$P(X = r) = K(n,r) \cdot p^r \cdot (1-p)^{n-r}$$

Mean μ (189)

$$\mu = n \cdot p$$

Standard deviation σ (190)

$$\sigma = \sqrt{n \cdot p \cdot (1-p)}$$

Number n of objects in the sample

95% confidence interval for population probability parameter p estimated from the sample proportion $\hat{p}$ (191)

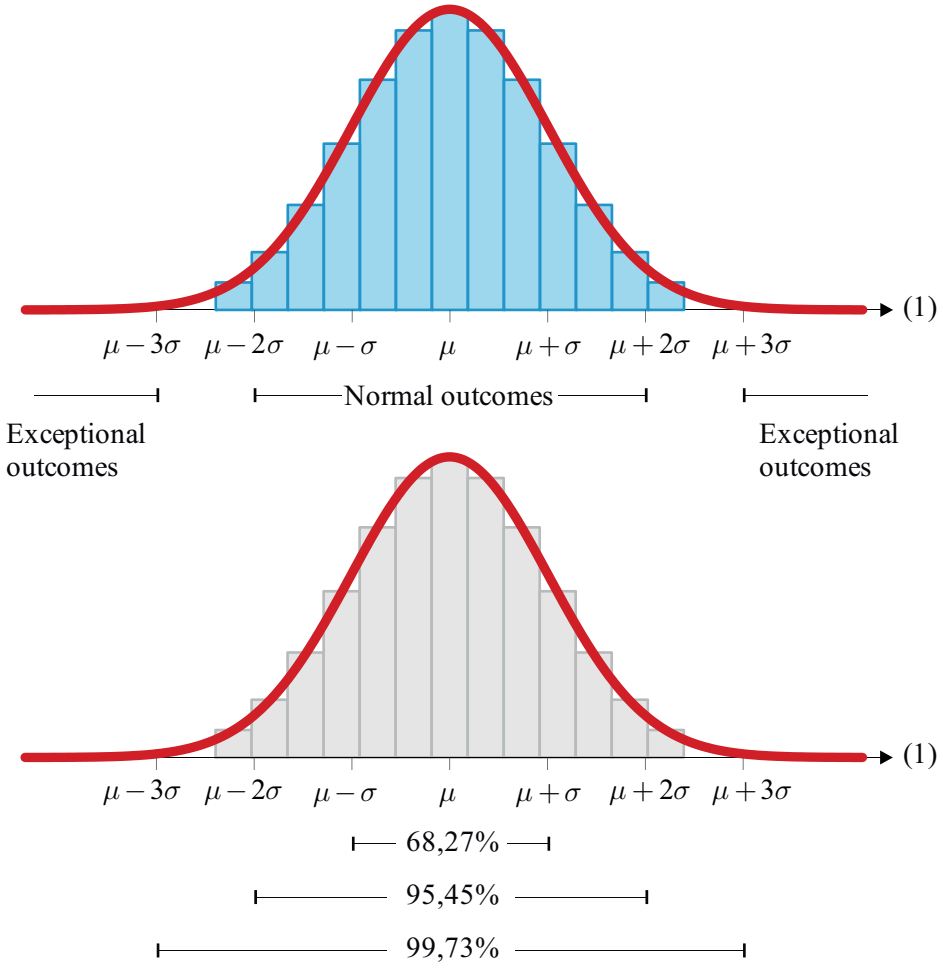
$$\left[\hat{p} - 2 \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}} \,;\, \hat{p} + 2 \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}\right]$$

Normal approximation to a binomially distributed random variable X with mean (192)

$$\mu = n \cdot p$$

and standard deviation

$$\sigma = \sqrt{n \cdot p \cdot (1-p)}$$



Pascal's triangle (193)

$K(0,0)$

$K(1,0)$ $K(1,1)$

$K(2,0)$ $K(2,1)$ $K(2,2)$

$K(3,0)$ $K(3,1)$ $K(3,2)$ $K(3,3)$

$K(4,0)$ $K(4,1)$ $K(4,2)$ $K(4,3)$ $K(4,4)$

$K(5,0)$ $K(5,1)$ $K(5,2)$ $K(5,3)$ $K(5,4)$ $K(5,5)$

$K(6,0)$ $K(6,1)$ $K(6,2)$ $K(6,3)$ $K(6,4)$ $K(6,5)$ $K(6,6)$

$K(7,0)$ $K(7,1)$ $K(7,2)$ $K(7,3)$ $K(7,4)$ $K(7,5)$ $K(7,6)$ $K(7,7)$

$K(8,0)$ $K(8,1)$ $K(8,2)$ $K(8,3)$ $K(8,4)$ $K(8,5)$ $K(8,6)$ $K(8,7)$ $K(8,8)$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

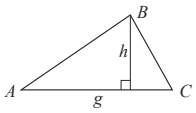
Multiplication table (194)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

Red numbers: Squares

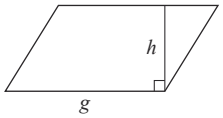
Area, circumference, volume and surface area of geometric figures

Triagle



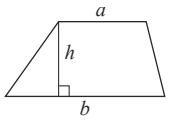
h	height	
g	base	
A	area	$A = \frac{1}{2} h \cdot g$

Parallelogram



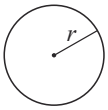
h	height	
g	base	
A	area	$A = h \cdot g$

Trapezium



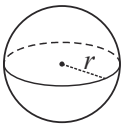
h	height	
a, b	parallel sides	
A	area	$A = \frac{1}{2} h \cdot (a + b)$

Circle



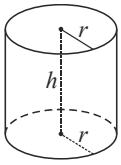
r	radius	
A	area	$A = \pi r^2$
O	circumference	$O = 2\pi r$

Sphere



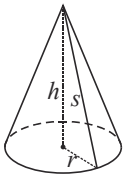
r	radius	
O	surface area	$O = 4\pi r^2$
V	volume	$V = \frac{4}{3} \pi r^3$

Cylinder



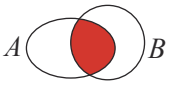
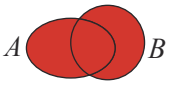
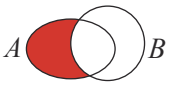
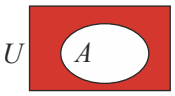
h	height	
r	base radius	
O	lateral surface area	$O = 2\pi r \cdot h$
V	volume	$V = \pi r^2 \cdot h$

Cone



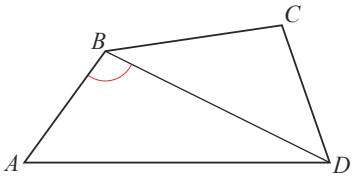
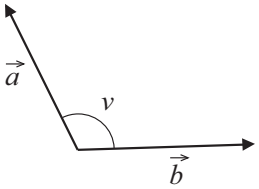
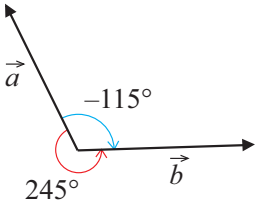
h	height	
s	length of slant	
r	base radius	
O	lateral surface area	$O = \pi r \cdot s$
V	volume	$V = \frac{1}{3} \pi r^2 \cdot h$

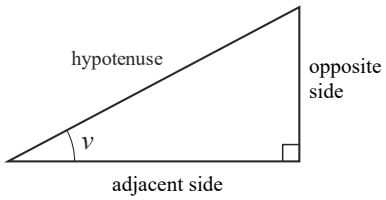
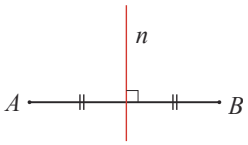
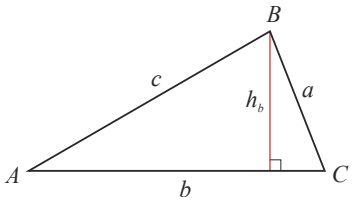
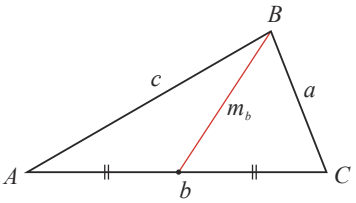
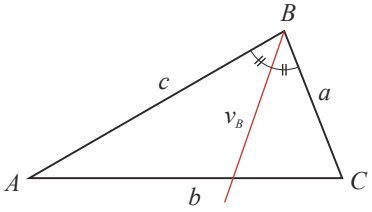
Mathematical symbols

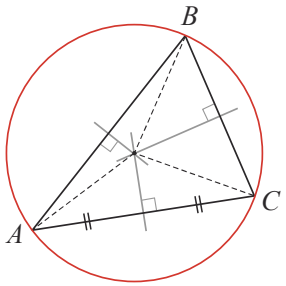
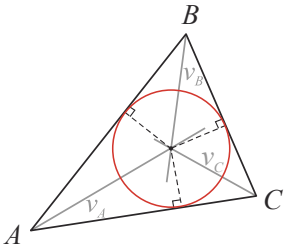
Symbol	Meaning	Examples, comments e.g.
$\{.,.,.,.\}$	set in list form	$\{-5,0,3,10\}, \{-5,0,3,10\}, \{2,4,6,...\}$
$\mathbb{N}$	the set of natural numbers	$\mathbb{N} = \{1,2,3,...\}$
$\mathbb{Z}$	the set of integers	$\mathbb{Z} = \{...,-2,-1,0,1,2,...\}$
$\mathbb{Q}$	the set of rational numbers	tal, der kan skrives $\frac{p}{q}$, $p \in \mathbb{Z}, q \in \mathbb{N}$
$\mathbb{R}$	the set of real numbers	
$\in$	belongs to / is an element of	$2 \in \mathbb{N}$
$[a\,;b]$	closed interval	$[1\,;3] = \{x \in \mathbb{R} \mid 1 \le x \le 3\}$
$]a\,;b]$	half-open interval	$]1\,;3] = \{x \in \mathbb{R} \mid 1 < x \le 3\}$
$[a\,;b[$	half-open interval	$[1\,;3[= \{x \in \mathbb{R} \mid 1 \le x < 3\}$
$]a\,;b[$	open interval	$]1\,;3[= \{x \in \mathbb{R} \mid 1 < x < 3\}$
$\subset$	is a proper subset of	$\{1,2,3\} \subset \mathrm{N}$
$\cap$	intersection set	$A \cap B$ 
$\cup$	union set	$A \cup B$ 
$\setminus$	set difference	$A \setminus B$ 
$\overline{A}$	complement of A	$U \setminus A$ 
$\emptyset$	the empty set	
	disjoint sets	$A \cap B = \emptyset$ 
$\times$	product set	$[-10;10] \times [-10;10]$
$\wedge$	”and” meaning ”both and” (conjunction)	$x < 2 \quad \wedge \quad y = 5$
$\vee$	”or” meaning ”and/or” (disjunction)	$x < 2 \quad \vee \quad x > 5$

Symbol	Meaning	Examples, comments e.g.
$\Rightarrow$	”implies”, ”if … then” (implication)	$x=2 \Rightarrow x^2=4$
$\Leftrightarrow$	”equivalent”, ”if and only if” (biconditional)	$x^2=4 \Leftrightarrow x=-2 \vee x=2$
$\sum_{i=1}^{n} a_i$	$a_1+a_2+\ldots+a_n$	$\sum_{i=1}^{4} i^2=1^2+2^2+3^2+4^2$
$n!$	n factorial	$n!=1\cdot 2\cdot \ldots \cdot n$ for $n\geq 1$ $0!=1$
$f(x)$	value of the function f at x	If $f(x)=\sqrt{2x+1}$, then $f(4)=3$.
$\mathrm{Dm}(f)$	domain of f	
$\mathrm{Vm}(f)$	range of f	
$f\circ g$	composite function	$(f\circ g)(x)=f(g(x))$
f^{-1}	inverse function	$s=f(t) \Leftrightarrow t=f^{-1}(s)$
$\log(x)$	logarithm with base 10	$y=\log(x) \Leftrightarrow x=10^y$
$\ln(x)$	natural logarithm	$y=\ln(x) \Leftrightarrow x=\mathrm{e}^y$
e^x	natural exponential function	e^x is also denoted $\exp(x)$
a^x	exponential function with base a, $a>0$	$b\cdot a^x$ can also be called an exponential function or exponential growth
x^a	power function	$b\cdot x^a$ can also be called a power function or power growth
$\lvert x\rvert$	absolute (numerical) value of x	$\lvert 3\rvert=3$, $\lvert -7\rvert=7$ $\lvert x\rvert$ is also denoted $\mathrm{abs}(x)$
$\sin(x)$	sine	
$\cos(x)$	cosine	
$\tan(x)$	tangent	$\tan(x)=\frac{\sin(x)}{\cos(x)}$

Symbol	Meaning	Examples, comments e.g.
$\sin^{-1}(y)$	inverse sine function	$\sin^{-1}(y)=x \Leftrightarrow \sin(x)=y$ $\sin^{-1}(0.5)=30°$ $\sin^{-1}$ is also denoted Arcsin
$\cos^{-1}(y)$	inverse cosine function	$\cos^{-1}(y)=x \Leftrightarrow \cos(x)=y$ $\cos^{-1}(0.5)=60°$ $\cos^{-1}$ is also denoted Arccos
$\tan^{-1}(y)$	inverse tangent function	$\tan^{-1}(y)=x \Leftrightarrow \tan(x)=y$ $\tan^{-1}(1)=45°$ $\tan^{-1}$ is also denoted Arctan
$\lim_{x\to x_0} f(x)$	limit of $f(x)$ as x tends to x_0	$\lim_{x\to 3}\sqrt{x+1}=2$
$\lim_{x\to\infty} f(x)$	limit of $f(x)$ as x tends to ∞	$\lim_{x\to\infty}\frac{1}{x}=0$
$f(x)\to a$ for $x\to x_0$	$f(x)$ tends to a as x tends to x_0	$\sqrt{x+1}\to 2$ for $x\to 3$
$f(x)\to a$ for $x\to\infty$	$f(x)$ tends to a as x tends to ∞	$e^{-x}\to 0$ for $x\to\infty$
Δx	change in x	$\Delta x = x - x_0$
$\Delta y, \Delta f$	change in $y=f(x)$	$\Delta y = \Delta f = f(x)-f(x_0)$
$\frac{\Delta y}{\Delta x}, \frac{\Delta f}{\Delta x}$	difference quotient of $y=f(x)$	$\frac{\Delta y}{\Delta x}=\frac{\Delta f}{\Delta x}=\frac{f(x)-f(x_0)}{x-x_0}$
$f'(x_0)$	derivative (differential quotient) of $y=f(x)$ at x_0	$f'(x_0)=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$ $=\lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$
f'	derivative of $y=f(x)$	denoted $f'(x),\ y',\ \frac{d}{dx}f(x),$ $\frac{d}{dx}(f(x)),\ \frac{df}{dx},\ \frac{dy}{dx},\ \left(\sqrt{3x^2+1}\right)'$
$f^{(n)}$	the n-th derivative of $y=f(x)$	$f^{(2)}(x)$ is often written $f''(x)$, y'' or $\frac{d^2y}{dx^2}$

Symbol	Meaning	Examples, comments e.g.
AB	line segment AB	
$\lvert AB \rvert$	length of the line segment AB	
$\overset{\frown}{AB}$	(circular) arc $\overset{\frown}{AB}$	
$\lvert \overset{\frown}{AB} \rvert$	length of the arc $\overset{\frown}{AB}$	
$\vec{a}, \overrightarrow{AB}$	vector	
$\lvert \vec{a} \rvert, \lvert \overrightarrow{AB} \rvert$	length (norm) of the vector	
$\hat{\vec{a}}$	perpendicular vector	the notation $\hat{a}$ can also be used
$\vec{a} \cdot \vec{b}$	scalar product, dot product	the notation $\vec{a} \cdot \vec{b}$ can also be used
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$	determinant of the pair of vectors $(\vec{a}, \vec{b})$	the notation $\det(\vec{a}, \vec{b})$ is also used
$\perp$	"is perpendicular to"	$l \perp m$ can also be read as "l and m are orthogonal"
$\angle A$	the angle A	$\angle A = 110°$ or $A = 110°$
$\angle ABD$	the angle B in triangle ABD	
$\angle(\vec{a}, \vec{b})$	angle v between $\vec{a}$ and $\vec{b}$, where $0° \le v \le 180°$	
	the angle from $\vec{a}$ to $\vec{b}$	

Symbol	Meaning	Examples, comments e.g.
	right-angled triangle	
	perpendicular bisector n of the line segment AB	
h_b	height from B on the side b or its extension	
m_b	median from B on the side b	
v_B	angular bisector of the angle B	

Symbol	Meaning	Examples, comments e.g.
	the circumscribed circle of triangle *ABC*	
	the inscribed circle of triangle *ABC*	

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